TRANSPORTATION

Flying

Problem Set Solutions

Problem 1: In flight Power Supply

Many airlines now provide electrical outlets to connect your laptop to in flight. Estimate how much more fuel a large airplane has to carry on a flight from Vancouver International Airport to London Heathrow Airport to accommodate for laptop use in flight. The fuel efficiency of jet engines is ~1/3.

Problem 1 Solution: In flight Power Supply

According to the Air Canada PC TravelDesk Utility, a flight from YVR (Vancouver) to LHR (London) is 7542 km and will take approximately 9 hours in a Boeing 777 (it could be longer or shorter depending on the wind). Using a Kill-a-Watt meter, a device that measures the amount of energy used by any appliance you plug into it, we found the average computer drew 65W. Also, according to Boeing, a 777-200 has 305 seats.

We will calculate the maximum amount of fuel a jet would have to carry in order to allow each passenger to run their laptops for the entire duration of the flight. This will be a substantial overestimation since not all passengers will have a laptop, most passengers will not plug their laptop in for the entire flight, and not all laptops will draw the same amount of power.

What we know:

Max # of computers = 305
Efficiency of a jet engine = 1/3
Max time the computers are on = 9 hours
Power a computer draws = 65 W
Energy content of jet fuel = 35 MJ/L

Fuel per passenger:

$$\text{Energy} = \left(\frac{65\,\text{J}}{\text{s}}\right)(9\,\text{h})\left(\frac{3600\,\text{s}}{1\,\text{h}}\right) = 2.1\,\text{x}\,10^6\ \text{J} = 2.1\,\text{MJ}$$

$$\text{Volume of Jet Fuel} = \frac{(2.1\,\text{MJ})\left(\frac{1\,\text{L}}{35\,\text{MJ}}\right)}{1/3} = 0.18\,\text{L}$$

For each passenger who plugs in their laptop for the entire duration of a 9 hour flight on a Boeing 777, 0.18 L additional jet fuel would be needed to provide this power.

If all 305 passengers did this, it would require 55 L. This is a small fraction of the fuel capacity of the 777 which is 117,340 L.

Problem 2: Fuel Economy

How does the fuel economy of a Boeing 747 compare to that of a 2010 Toyota Prius (in L/100 km/passenger)?

Problem 2 Solution: Fuel Economy

A Boring 747-8 has a maximum distance of ~ 15, 000 km, a fuel capacity of 240,000 L and has a passenger capacity of 400-500.

$$\text{Fuel Economy} = \frac{240{,}000\text{ L}}{15{,}000\text{ km}} = 16\text{ L/km}$$

The fuel economy per person per 100 km:

$$\frac{16\text{ L/km}}{400\text{ people}} = 0.04\text{ L/km/person} = 4\text{ L/100km/person}$$

So, the consumption per passenger is 4.0 L/100 km when the plane has 400 passengers and 3.2 L/100 km when the plane has 500 passengers.

For a 2010 Toyota Prius, the fuel economy is 4.7 L/100 km. Given that the average number of passengers is 1.5, the fuel efficiency is

$$\frac{4.7\text{ L/100 km}}{1.5\text{ people}} = 3.1\text{ L/100km/person}$$

So the fuel efficiency of a Toyota Prius with 1.5 people in it is actually quite similar to that of a full 747. So, if you are travelling anywhere with more than 1.5 people, drive!

Problem 3: Fuel Demands

How much fuel does a plane require to go from rest on the runway to the cruising speed at their typical altitude? Where does the rest of the fuel go into? How much fuel does it cost the plane to fly for the same amount of time at the cruising altitude and speed? Should planes fly at higher or lower altitudes?

Problem 3 Solution: Fuel Demands

A Boeing 747 has a mass of 440,000 kg at take off, a cruising speed of 290 m/s, a cruising altitude of 10.5 km and an efficiency of its jet engines of 1/3.

The energy required to get it to the airplane cruising:

$$
\begin{aligned}
\text{Energy} &= PE + KE \\
&= mgh + \frac{1}{2}mv^2 \\
&= (440{,}000\ \text{kg})(9.8\text{m/s}^2)(10{,}500\ \text{m}) + \frac{1}{2}(440{,}000\ \text{kg})(290\ \text{m/s})^2 \\
&= 64\ \text{GJ} \\
&= 192\ \text{GJ when taking an efficiency of 1/3 into account}
\end{aligned}
$$

The fuel required:

$$\text{Fuel} = (192\text{x}10^9\ \text{J})(\frac{1\ \text{L}}{35\text{x}10^6\ \text{J})} = 5500\ \text{L}$$

The energy cost is multiplied by 3 to account for the efficiency of the engine. The fuel required to accelerate and elevate the airplane is 5500 L. Given that the fuel capacity of the plane is 240,000 L, this means that ~230,000 L of fuel goes into maintaining this speed against air drag.

How much fuel is required to cruise at cruising speed and altitude? We will assume that it takes 20 min for the plane to get to cruising speed and attitude. When in flight, the airplane just has to overcome the drag force (note: the area for calculating the drag force of an airplane is the wing area, not the cross-sectional area, as it is when dealing with cars).

$$
\begin{aligned}
\text{F}_\text{D} &= \frac{1}{2}\rho v^2 C_D A \\
&= \frac{1}{2}(0.4\ \text{kg/m}^3)(290\ \text{m/s})^2(0.031)((525\ \text{m}^2) \\
&= 274{,}000\ \text{N}
\end{aligned}
$$

In 20 min, the plane travels

$$(20\text{min})(\frac{60\ \text{s}}{1\ \text{min}})(290\ \text{m/s}) = 348{,}000\ \text{m}$$

Putting this together and taking efficiency into account:

$$
\begin{aligned}
W &= Fd \\
&= 3(274{,}000\ \text{N})(348{,}000\ \text{m}) \\
&= 285\ \text{GJ}
\end{aligned}
$$

The fuel required:

$$\text{Fuel} = (285\text{x}10^9\ \text{J})(\frac{1\ \text{L}}{35\text{x}10^6\ \text{J})} = 8200\ \text{L}$$

Getting up to cruising altitude and speed requires less energy than it does to maintain the cruising altitude and speed. From this we see that climbing to a higher altitude requires little fuel, and at a higher altitude there is a decreased air density, leading to decreased air drag, requiring less fuel than flying at a lower altitude. Therefore, planes should fly at higher altitudes. Or should they? Is it better or worse for the environment to fly at high altitudes? Check out the next question!

Note: In optimal flight conditions, the induced drag (from lift) is 1/3 of that from parasitic drag (what we just calculated), and so the total drag is 4/3 of our calculation. For more information see http://en.wikipedia.org/wiki/Lift-induced_drag.

Problem 4: Greenhouse Gas Emissions

Above 6,000 or 7,000 m, there is very little water vapour in the atmosphere. Airplanes are powered by burning jet fuel which is roughly given by $CH_2 + {}^3/_2O_2 \rightarrow CO_2 + H_2O$. How much water is produced in the longest flight a 747 will make and what effect does this water have on the atmosphere?

Problem 4 Solution: Greenhouse Gas Emissions

The mass of jet fuel used: (240,000 L)(0.81 kg/L) = 194,400 kg

$$CH_2\ (14\ g/mol) + {}^3/_2O_2\ (32\ g/mol) \rightarrow CO_2\ (44\ g/mol) + H_2O\ (18\ g/mol)$$

Using molecular masses, we can see that 14 kg of gas will produce 18 kg of water, so using 194,400 kg of gas will produce (194,400 kg)(18/14) = 250,000 kg of water.

So, over the course of a long flight, 250 tonnes of water is added to the atmosphere where there normally isn't any. As the hot exhaust cools, it forms a trail of condensation, called contrails, which are artificial clouds can exist for several hours. These contrails affect the earth's radiation balance by trapping outgoing radiation and therefore warming the earth. The effect of this is larger at night, due to the wavelengths of the radiation, and while only 25% of air traffic occurs at night, this accounts for 82% of the effect of contrails on the radiation balance. Also, winter flights are twice as likely to form contrails.

Natural clouds also affect the earth's radiation balance as they reflect more sunlight than the surface of the Earth. The effect of clouds is difficult to calculate because clouds can both keep the earth cool by reflecting incoming sunlight and heat up the Earth by trapping the heat that penetrates them, depending on the height and thickness of the cloud. For more information on this, see Kump, L.R., Kasting, J.F., and Crane, R.G. The Atmospheric Circulation System. In: The Earth System (2), edited by Patrick Lynch. Upper Saddle River, New Jersy, USA: 2004, chapt. 10, pp. 273 - 276.

Brittany Tymos (July 29, 2010)

References:

MacKay DJC. Sustainable Energy - Without the Hot Air. *Planes II* (online). UIT Cambridge. p. 282. http://www.inference.phy.cam.ac.uk/withouthotair/cC/page_282.shtml [20 July 2010].

Boeing. 777 Interior Arrangements (online).
http://www.boeing.com/commercial/777family/pf/pf_seating_charts.html [20 *July 2010].*

Wikipedia. *Fuel Efficiency in Transportation* (online).
http://en.wikipedia.org/wiki/Fuel_efficiency_in_transportation [21 July 2010].

Wikipedia. *Toyota Prius* (online). http://en.wikipedia.org/wiki/Toyota_Prius [21 July 2010].

Boeing. *747-8 Technical Characteristics* (online). http://www.boeing.com/commercial/747family/747-8_fact_sheet.html [21 July 2010].

The Engineering Toolbox. *U.S Standard Atmosphere Air Properties in SI Units* (online).
http://www.engineeringtoolbox.com/standard-atmosphere-d_604.html [11 Aug 2010].

Top Speed. *2011 Boeing 747-8* (online). http://www.topspeed.com/aviation/aviation-reviews/boeing/2011-boeing-747-8-ar86257.html [1 June 2010].

Wikipedia. *Contrail* (online). http://en.wikipedia.org/wiki/Contrail [27 July 2010].

Stuber, N., Forster, P., Radel, G., & Shine, K.. *The Importance of the Diurnal and Annual Cycle of Air Traffic for Contrail Radiative Forcing.* Nature:441, pg 864-867.

Wikipedia. *Albedo* (online). http://en.wikipedia.org/wiki/Albedo [3 August 2010].